

NAG C Library Function Document

nag_dppcon (f07ggc)

1 Purpose

nag_dppcon (f07ggc) estimates the condition number of a real symmetric positive-definite matrix A , where A has been factorized by nag_dpptrf (f07gdc), using packed storage.

2 Specification

```
void nag_dppcon (Nag_OrderType order, Nag_UploType uplo, Integer n,
                 const double ap[], double anorm, double *rcond, NagError *fail)
```

3 Description

nag_dppcon (f07ggc) estimates the condition number (in the 1-norm) of a real symmetric positive-definite matrix A :

$$\kappa_1(A) = \|A\|_1 \|A^{-1}\|_1.$$

Since A is symmetric, $\kappa_1(A) = \kappa_\infty(A) = \|A\|_\infty \|A^{-1}\|_\infty$.

Because $\kappa_1(A)$ is infinite if A is singular, the function actually returns an estimate of the **reciprocal** of $\kappa_1(A)$.

The function should be preceded by a call to nag_dsp_norm (f16rdc) to compute $\|A\|_1$ and a call to nag_dpptrf (f07gdc) to compute the Cholesky factorization of A . The function then uses Higham's implementation of Hager's method (see Higham (1988)) to estimate $\|A^{-1}\|_1$.

4 References

Higham N J (1988) FORTRAN codes for estimating the one-norm of a real or complex matrix, with applications to condition estimation *ACM Trans. Math. Software* **14** 381–396

5 Parameters

1: **order** – Nag_OrderType *Input*

On entry: the **order** parameter specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order = Nag_RowMajor**. See Section 2.2.1.4 of the Essential Introduction for a more detailed explanation of the use of this parameter.

Constraint: **order = Nag_RowMajor** or **Nag_ColMajor**.

2: **uplo** – Nag_UploType *Input*

On entry: indicates whether A has been factorized as $U^T U$ or LL^T as follows:

if **uplo = Nag_Upper**, $A = U^T U$, where U is upper triangular;

if **uplo = Nag_Lower**, $A = LL^T$, where L is lower triangular.

Constraint: **uplo = Nag_Upper** or **Nag_Lower**.

3: **n** – Integer *Input*

On entry: n , the order of the matrix A .

Constraint: $n \geq 0$.

- 4: **ap**[*dim*] – const double *Input*
Note: the dimension, *dim*, of the array **ap** must be at least $\max(1, \mathbf{n} \times (\mathbf{n} + 1)/2)$.
On entry: the Cholesky factor of *A* stored in packed form, as returned by nag_dpptf (f07gdc).
- 5: **anorm** – double *Input*
On entry: the 1-norm of the **original** matrix *A*, which may be computed by calling nag_dsp_norm (f16rdc). **anorm** must be computed either **before** calling nag_dpptf (f07gdc) or else from a copy of the original matrix *A*.
Constraint: **anorm** ≥ 0.0 .
- 6: **rcond** – double * *Output*
On exit: an estimate of the reciprocal of the condition number of *A*. **rcond** is set to zero if exact singularity is detected or the estimate underflows. If **rcond** is less than *machine precision*, *A* is singular to working precision.
- 7: **fail** – NagError * *Output*
The NAG error parameter (see the Essential Introduction).

6 Error Indicators and Warnings

NE_INT

On entry, **n** = *<value>*.
Constraint: **n** ≥ 0 .

NE_REAL

On entry, **anorm** = *<value>*.
Constraint: **anorm** ≥ 0.0 .

NE_ALLOC_FAIL

Memory allocation failed.

NE_BAD_PARAM

On entry, parameter *<value>* had an illegal value.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

7 Accuracy

The computed estimate **rcond** is never less than the true value ρ , and in practice is nearly always less than 10ρ , although examples can be constructed where **rcond** is much larger.

8 Further Comments

A call to nag_dppcon (f07ggc) involves solving a number of systems of linear equations of the form $Ax = b$; the number is usually 4 or 5 and never more than 11. Each solution involves approximately $2n^2$ floating-point operations but takes considerably longer than a call to nag_dpptf (f07gdc) with 1 right-hand side, because extra care is taken to avoid overflow when *A* is approximately singular.

The complex analogue of this function is nag_zppcon (f07guc).

9 Example

To estimate the condition number in the 1-norm (or infinity-norm) of the matrix A , where

$$A = \begin{pmatrix} 4.16 & -3.12 & 0.56 & -0.10 \\ -3.12 & 5.03 & -0.83 & 1.18 \\ 0.56 & -0.83 & 0.76 & 0.34 \\ -0.10 & 1.18 & 0.34 & 1.18 \end{pmatrix}.$$

Here A is symmetric positive-definite, stored in packed form, and must first be factorized by `nag_dpptrf` (f07gdc). The true condition number in the 1-norm is 97.32.

9.1 Program Text

```
/* nag_dppcon (f07ggc) Example Program.
 *
 * Copyright 2001 Numerical Algorithms Group.
 *
 * Mark 7, 2001.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagf07.h>
#include <nagf16.h>
#include <nagx02.h>

int main(void)
{
    /* Scalars */
    double anorm, rcond;
    Integer ap_len, i, j, n;
    Integer exit_status=0;
    NagError fail;
    Nag_UploType uplo_enum;
    Nag_OrderType order;

    /* Arrays */
    char uplo[2];
    double *ap=0;

#ifdef NAG_COLUMN_MAJOR
#define A_UPPER(I,J) ap[J*(J-1)/2 + I - 1]
#define A_LOWER(I,J) ap[(2*n-J)*(J-1)/2 + I - 1]
    order = Nag_ColMajor;
#else
#define A_LOWER(I,J) ap[I*(I-1)/2 + J - 1]
#define A_UPPER(I,J) ap[(2*n-I)*(I-1)/2 + J - 1]
    order = Nag_RowMajor;
#endif

    INIT_FAIL(fail);
    Vprintf("f07ggc Example Program Results\n\n");
    /* Skip heading in data file */
    Vscanf("%*[\n] ");
    Vscanf("%ld%*[\n] ", &n);
    ap_len = n * (n + 1)/2;

    /* Allocate memory */
    if ( !(ap = NAG_ALLOC(ap_len, double)) )
    {
        Vprintf("Allocation failure\n");
        exit_status = -1;
        goto END;
    }

    /* Read A from data file */
    Vscanf(" ' %1s '%*[\n] ", uplo);
```

```

if (*(unsigned char *)uplo == 'L')
    uplo_enum = Nag_Lower;
else if (*(unsigned char *)uplo == 'U')
    uplo_enum = Nag_Upper;
else
{
    Vprintf("Unrecognised character for Nag_UploType type\n");
    exit_status = -1;
    goto END;
}
if (uplo_enum == Nag_Upper)
{
    for (i = 1; i <= n; ++i)
    {
        for (j = i; j <= n; ++j)
            Vscanf("%lf", &A_UPPER(i,j));
    }
    Vscanf("%*[^\\n] ");
}
else
{
    for (i = 1; i <= n; ++i)
    {
        for (j = 1; j <= i; ++j)
            Vscanf("%lf", &A_LOWER(i,j));
    }
    Vscanf("%*[^\\n] ");
}

/* Compute norm of A */
f16rdc(order, Nag_OneNorm, uplo_enum, n, ap, &anorm, &fail);
if (fail.code != NE_NOERROR)
{
    Vprintf("Error from f16rdc.\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}
/* Factorize A */
f07gdc(order, uplo_enum, n, ap, &fail);
if (fail.code != NE_NOERROR)
{
    Vprintf("Error from f07gdc.\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}
/* Estimate condition number */
f07ggc(order, uplo_enum, n, ap, anorm, &rcond, &fail);
if (fail.code != NE_NOERROR)
{
    Vprintf("Error from f07ggc.\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}
if (rcond >= X02AJC)
    Vprintf("Estimate of condition number =%10.2e\\n\\n", 1.0/rcond);
else
    Vprintf("A is singular to working precision\\n");
END:
if (ap) NAG_FREE(ap);
return exit_status;
}

```

9.2 Program Data

f07ggc Example Program Data

```
4                               :Value of N
'L'                             :Value of UPLO
4.16
-3.12    5.03
0.56    -0.83    0.76
-0.10    1.18    0.34    1.18    :End of matrix A
```

9.3 Program Results

f07ggc Example Program Results

Estimate of condition number = 9.73e+01
